Dielectrics

Dielectrics, insulating materials placed between the plates of a capacitor, cause the electric field inside the capacitor to be reduced for the same amount of charge on the plates. This is because the molecules of the dielectric material get polarized in the field, and they align themselves in a way that sets up another field inside the dielectric opposite to the field from the capacitor plates. The dielectric constant is the ratio of the electric field without the dielectric to the field with the dielectric:

$$\text{dielectric constant} : \kappa = E_0 / E$$

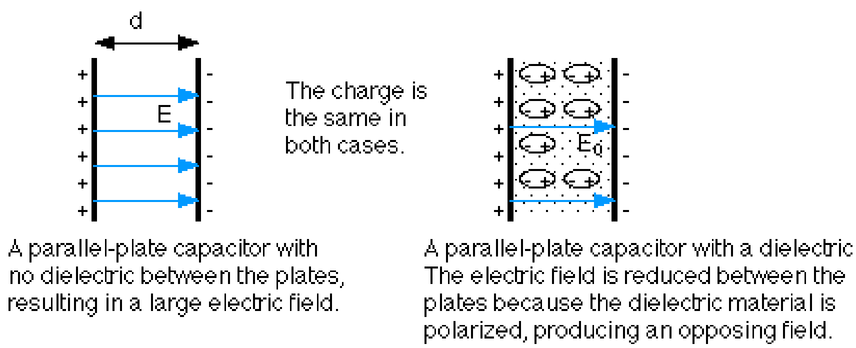


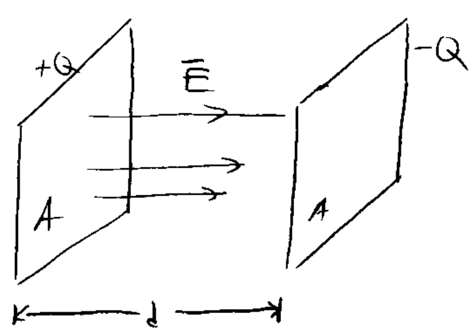
Figure 3.4

Being a ratio of two field strengths, the dielectric constant is a number without units. Moreover, since the field E_0 without the dielectric is greater than the field E inside the dielectric, the dielectric constant is greater than unity. The value of *K* depends on the nature of the dielectric material.

Capacitance of a Plane Capacitor

Example two parallel conducting plates.

Assume that that the E-field is perpendicular to the plates, all over the plates. This means we are neglecting edge-effects, where the field lines curve outward past the edges of the plates. We might also say that we are assuming that the two plates are infinite in area. In such a case, the E-field is uniform between the plates.



$$\varphi = EA = \frac{Q}{\varepsilon_o}$$

$$E = \frac{Q}{A\varepsilon_o}$$

$$\Delta V = Ed \Rightarrow E = \frac{V}{d}$$

Set the two expressions for the E-field equal, and solve for $\frac{Q}{V}$.

$$\frac{V}{d} = \frac{Q}{A\varepsilon_o}$$

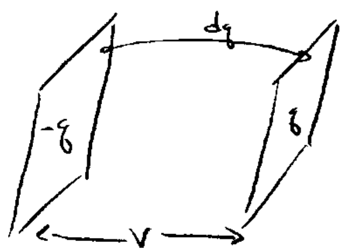
$$\frac{Q}{V} = \frac{A\varepsilon_o}{d} = C$$

Notice that the capacitance of the two parallel plates depends on the geometry of the plates--area and separation.

Energy storage

In order to separate the $+Q$ and $-Q$ charges on the two plates, work must be done. Let's say we transfer a small amount of charge, dq from one plate to another that already has an excess charge q on it. The work done is $dW = Vdq$. To go from 0 to Q, the total work is

$$W = \int_0^Q Vdq = \int_0^Q \frac{q}{C}dq = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}QV = U$$

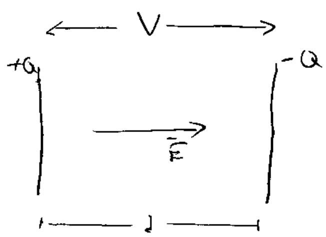


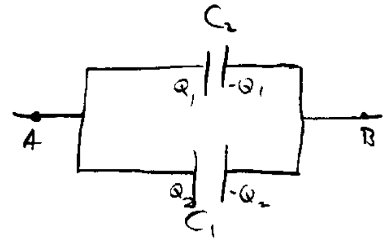
This is a change in electrostatic potential energy, which we regard as being stored on or in the capacitor.

It is convenient to associate the stored energy with the electric field that "fills" the space between the plates. We define the *energy density* to be the energy per unit volume.

$$u = \frac{U}{Ad} = \frac{\frac{1}{2}CV^2}{Ad} = \frac{\frac{1}{2}\left(\frac{\varepsilon_o A}{d}\right)E^2 d^2}{Ad} = \frac{1}{2}\varepsilon_o E^2$$

Dielectric breakdown in a capacitor





The constant ε_o is the permitivity of the vacuum, or of "free space." Material substances, particularly insulators, called also *dielectric* materials, have permitivities greater than ε_o. If the space between the plates is filled with an insulating substance, then the capacitance is increased. $C = \frac{\varepsilon A}{d}$. More charge can be stored at the same voltage. However, the E-field polarizes the molecules of the insulating substance. If the E-field is large enough, it ionizes those molecules, and the substance becomes conductive, leading to a spark discharge. The dielectric has *broken down*.

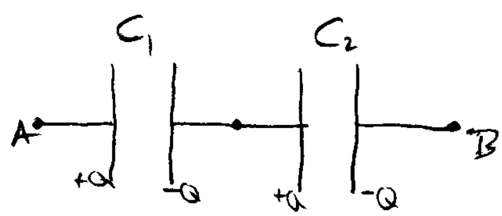
Combinations of capacitors

i) series

For capacitors connected in series, the charge on each capacitor is the same *Q*. If it weren't, charge would flow until it was.

$$V_{AB} = V_1 + V_2$$

$$V_{AB} = \frac{Q}{C_1} + \frac{Q}{C_2} = Q\left(\frac{1}{C_1} + \frac{1}{C_2}\right) = \frac{Q}{C_{eq}}$$



The equivalent capacitance of the series combination is $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots$.

ii) parallel

For capacitors connected in parallel, it's the voltage that is the same on each of the capacitors.

$$V_{AB} = V_1 = V_2 = \frac{Q_1}{C_1} = \frac{Q_2}{C_2}$$

The total charge stored on the capacitors is $Q = Q_1 + Q_2$. Therefore,

$$V_{AB} = \frac{Q}{C_1 + C_2} = \frac{Q}{C_{eq}}$$
$$C_{eq} = C_1 + C_2 + \cdots$$

Sing it out: "capacitors in series have the same charge; capacitors in parallel have the same voltage."

Example: The capacitance of an empty capacitor is 1.2 μF. The capacitor is connected to a 12 V battery and charged up. With the capacitor connected to the battery, a slab of dielectric material is inserted between the plates. As a result, 2.6×10^{-5} C of *additional* charge flows from one plate, through the battery, and onto the other plate. What is the dielectric constant of the material?

$Q_0 = C_0V$, $Q_d = C_d V$, $K = C_d / C_0$

The Physics of a Computer Keyboard

One kind of computer keyboard is based on the idea of capacitance. Each key is mounted on one end of a plunger, and the other end is attached to a movable metal plate (see Figure 19.19). The movable plate is separated from a fixed plate, the two plates forming a capacitor. When the key is pressed, the movable plate is pushed closer to the fixed plate, and the capacitance increases. Electronic circuitry enables the computer to detect the *change* in capacitance, thereby recognizing which key has been pressed. The separation of the plates is normally 5.00×10^{-3} m but decreases to 0.150×10^{-3} m when a key is pressed. The plate area is 9.50×10^{-5} m^2, and the capacitor is filled with a material whose dielectric constant is 3.50. Determine the change in capacitance that is detected by the computer.

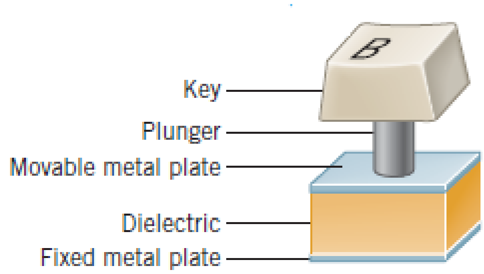


Figure 19.19 In one kind of computer keyboard, each key, when pressed, changes the separation between the plates of a capacitor.

Solution We can use Equation 19.10 directly to find the capacitance of the key, since the dielectric constant K, the plate area A, and the plate separation d are known. We will use this relation twice, once to find the capacitance when the key is pressed and once when it is not pressed. The change in capacitance will be the difference between these two values.

When the key is pressed, the capacitance is

$$C = \frac{\kappa\epsilon_0 A}{d} = \frac{(3.50)[8.85 \times 10^{-12}\ C^2/(N\cdot m^2)](9.50 \times 10^{-5}\ m^2)}{0.150 \times 10^{-3}\ m}$$

$$= 19.6 \times 10^{-12}\ F \quad (19.6\ pF)$$

A calculation similar to the one above reveals that when the key is *not* pressed, the capacitance has a value of 0.589×10^{-12} F (0.589 pF). The *change* in capacitance is an increase of The *change* in the capacitance is greater with the dielectric present, which makes it easier for the circuitry within the computer to detect it.

Example 2: In Fig. 25-7*a*, switch S is closed to connect the uncharged capacitor of capacitance C = 0.25 μF to the battery of potential difference V = 12 V.The lower capacitor plate has thickness L = 0.50 cm and face area $A = 2.0 \times 10^{-4}\ m^2$, and it consists of copper, in which the density of conduction electrons is $n = 8.49 = 10^{28}$ electrons/m^3. From what depth d within the plate (Fig. 25-7*b*) must electrons move to the plate face as the capacitor becomes charged?

Key idea: The charge collected on the plate is related to the capacitance and the potential difference across the capacitor by Eq: ($q = CV$).

Calculations: Because the lower plate is connected to the negative terminal of the battery, conduction electrons move up to the face of the plate. From Eq. 25-1, the total charge magnitude that collects there is:

$$q = CV = (0.25\times10^{-6}F)(12V) = 3.0 \times10^{-6}\ C$$

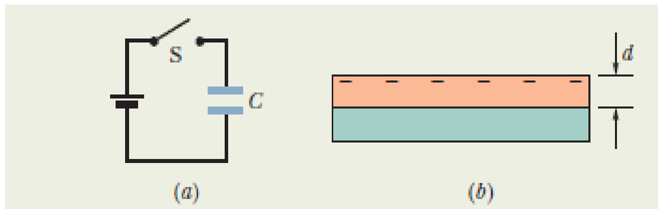


Fig. 25-7 (*a*) A battery and capacitor circuit. (*b*) The lower capacitor plate.

Dividing this result by e gives us the number N of conduction electrons that come up to the face:

$$N = \frac{q}{e} = \frac{3.0 \times 10^{-6}\ C}{1.602 \times 10^{-19}\ C} = 1.873 \times 10^{13}\ \text{electrons.}$$

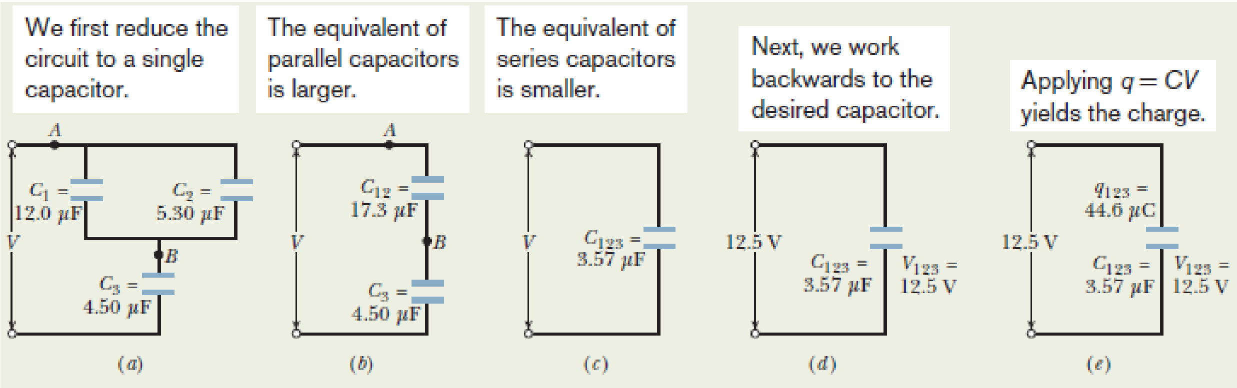
These electrons come from a volume that is the product of the face area A and the depth d we seek.Thus, from the density of conduction electrons (number per volume), we can write:

$$n = \frac{N}{Ad}, \quad \text{or} \quad d = \frac{N}{An} = \frac{1.873 \times 10^{13}\ \text{electrons}}{(2.0 \times 10^{-4}\ m^2)(8.49 \times 10^{28}\ \text{electrons}/m^3)}$$

$$= 1.1 \times 10^{-12}\ m = 1.1\ pm. \qquad \text{(Answer)}$$

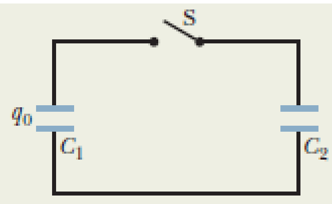
In common speech, we would say that the battery charges the capacitor by supplying the charged particles. But what the battery really does is set up an electric field in the wires and plate such that electrons very close to the plate face move up to the negative face.

Example 3: Find the equivalent capacitance for the combination of capacitances shown in Fig. 25-10*a*, across which potential difference *V* is applied $V = 12.5$V. Find the charge *q*



Example 4: Capacitor 1, with C_1 = 3.55 μF, is charged to a potential difference V_0 = 6.30 V, using a 6.30 V battery.The battery is then removed, and the capacitor is connected as in Fig. 25-11 to an uncharged capacitor 2, with C_2 = 8.95 μF.When switch S is closed, charge flows between the capacitors. Find the charge on each capacitor when equilibrium is reached.

Fig. 25-11 A potential difference V_0 is applied to capacitor 1 and the charging battery is removed. Switch S is then closed so that the charge on capacitor 1 is shared with capacitor 2.



Initially, when capacitor 1 is connected to the battery, the charge it acquires is, from;

$$q_0 = C_1V_0 = (3.55 \times 10^{-6}\,\text{F})\,(6.30\,\text{V}) \;\; = 22.365 \times 10^{-6}\,\text{C}.$$

When switch S in Fig. 25-11 is closed and capacitor 1 begins to charge capacitor 2, the electric potential and charge on capacitor 1 decrease and those on capacitor 2 increase until $V_1 = V_2$

we can rewrite this as
$$\frac{q_1}{C_1} = \frac{q_2}{C_2} \qquad \text{(equilibrium).}$$
Because the total charge cannot magically change, the total after the transfer must be $q_1 = q_2 = q_0$ (charge conservation);

thus $q_2 = q_0 - q_1.$

We can now rewrite the second equilibrium equation as

$$\frac{q_1}{C_1} = \frac{q_0 - q_1}{C_2}.$$

Solving this for q_1 and substituting given data, we find

$$q_1 = 6.35\,\mu\text{C}. \qquad \text{(Answer)}$$

The rest of the initial charge ($q_0 = 22.365\,\mu$C) must be on capacitor 2:

$$q_2 = 16.0\,\mu\text{C}. \qquad \text{(Answer)}$$